

Parallel Lepton Mass Matrices with Texture/Cofactor Zeros

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Abstract

In this paper we propose the parallel texture structures with texture zeros in the charged lepton mass matrix M_l and cofactor zeros in neutrino mass matrix M_ν . Using the weak basis permutation transformation, the 15 parallel textures can be grouped as 4 classes (class I,II,III and IV) where the matrices in each class share the same physical implications. The matrices in class IV is not phenomenological viable because one generation of the leptons decouples from the mixing. Among the other three classes, only the class II is phenomenologically acceptable for normal hierarchy by the newest 3σ global fit data. The correlations of some important physical variables are presented, which are essential for the model selection and can be potentially test by future experiments. The model realization of class II by means of $Z_4 \times Z_2$ flavor symmetry is illustrated.

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I. INTRODUCTION

The understanding of the leptonic flavor structure is one of the major open questions in particle physics. Several attempts have been proposed to explain the origin of neutrino mass and the observed pattern of leptonic mixing by introducing the flavor symmetries within the framework of seesaw models. The flavor symmetry often reduces the number of free parameters and leads to the specific structures of fermion mass matrices including texture zeros[1], hybrid textures[3, 4], zero trace[5], zero determinant[6], vanishing minors[8, 9], two traceless submatrices[10], equal elements or cofactors[11], hybrid M_ν^{-1} textures[12]. Among these models, the matrices with texture/cofactor zeros are particularly interesting due to their connections to the flavor symmetries. The phenomenological analyses of texture/cofactor zeros conditions in flavor basis have been widely studies in Ref.[1, 2, 7–9] where the charged lepton mass matrices M_l is diagonal. However, no priori reason is required that the flavor basis is necessary and the more general cases should be considered in nodiagonal basis. In this scenario, the systematic analysis of the fermion mass matrices with texture zeros in both M_l and neutrino mass matrix M_ν are investigated by many authors[7–9].

In this work, we propose a new possible mass matrices where there are two texture zeros in M_l and two cofactor zeros in M_ν and we study their phenomenological implications. It seems that such mass matrices are unusual because one instinctively expects the type of texture structures to be the same for M_l and M_ν . However, one reminds the type-I seesaw model as $M_\nu = -M_D M_R^{-1} M_D^T$. Then the texture or cofactor zeros of M_ν can be attributed to the texture zeros in M_D and M_R . Generally, this can be realized by Z_n flavor symmetry[7, 14]. Therefore from the point of flavor symmetry, both texture zeros and cofactor zeros structures manifest just the same flavor symmetry in different ways. It is the main motivation of the paper and we will build a concrete model in the following section. Furthermore, in order to simplify the analysis we take the so-called the parallel *Ansätze* where the positions of texture zeros in M_l and cofactor zeros in M_ν are the same. The parallel texture structures are

usually regarded in many literatures as the precursor of the more general cases. The leptonic mass matrices with parallel texture zero structures have been systematically investigated in Ref.[15]. Subsequently, the idea is generalized to more complicated situations such as parallel hybrid textures[16], parallel cofactor zero textures[17]. In our case, there exists $C_6^2 = 15$ logically possible patterns for two texture/cofactor zeros in mass matrices. It is indicated that the 15 textures can be grouped into 4 classes with the matrices in each classes connected by S_3 permutation matrices and sharing the same physical implications. Among the 4 classes, it is founded that one of the classes is not viable phenomenologically. Therefore we focus on the other three nontrivial classes.

The paper is organized as follow. In Sec. II, we present the classification of the matrices and relate the texture/cofactor zero condition to the newest experimental results. In Sec. III, we diagonalize the mass matrices, confront the numerical results with the experimental data and discuss their predictions. In Sec. IV, the flavor symmetry realization is given. We summarize our results in Sec. V.

II. FORMALISM

A. Weak basis equivalent classes

As shown in Ref.[15], the most general WB transformations leaving gauge currents invariant is given by

$$M_l \rightarrow M'_l = W^\dagger M_l W_R \quad M_\nu \rightarrow M'_\nu = W^T M_\nu W \quad (1)$$

where the neutrinos are assumed to be Majorana fermions and W , W_R are 3×3 unitary matrices. Two matrices related by WB transformations have the same physical implications. Therefore the parallel matrices with texture/cofactor zeros located at different positions can be connected by S_3 permutation matrix P as a specific WB transformation

$$M'_l = P^T M_l P \quad M'_\nu = P^T M_\nu P \quad (2)$$

It is noted that P changes the positions of cofactor zero elements but still preserves the parallel structures for both charged lepton and neutrino mass textures. Then the texture/cofactor zeros matrices are classified into 4 classes:

Class I:

$$\begin{pmatrix} 0/\Delta & \times & 0/\Delta \\ \times & \times & \times \\ 0/\Delta & \times & \times \end{pmatrix} \quad \begin{pmatrix} 0/\Delta & 0/\Delta & \times \\ 0/\Delta & \times & \times \\ \times & \times & \times \end{pmatrix} \quad \begin{pmatrix} \times & 0/\Delta & \times \\ 0/\Delta & 0/\Delta & \times \\ \times & \times & \times \end{pmatrix} \\ \begin{pmatrix} \times & \times & \times \\ \times & 0/\Delta & 0/\Delta \\ \times & 0/\Delta & \times \end{pmatrix} \quad \begin{pmatrix} \times & \times & 0/\Delta \\ \times & \times & \times \\ 0/\Delta & \times & 0/\Delta \end{pmatrix} \quad \begin{pmatrix} \times & \times & \times \\ \times & \times & 0/\Delta \\ \times & 0/\Delta & 0/\Delta \end{pmatrix} \quad (3)$$

Class II:

$$\begin{pmatrix} 0/\Delta & \times & \times \\ \times & \times & 0/\Delta \\ \times & 0/\Delta & \times \end{pmatrix} \quad \begin{pmatrix} \times & \times & 0/\Delta \\ \times & 0/\Delta & \times \\ 0/\Delta & \times & \times \end{pmatrix} \quad \begin{pmatrix} \times & 0/\Delta & \times \\ 0/\Delta & \times & \times \\ \times & \times & 0/\Delta \end{pmatrix} \quad (4)$$

Class III:

$$\begin{pmatrix} 0/\Delta & \times & \times \\ \times & 0/\Delta & \times \\ \times & \times & \times \end{pmatrix} \quad \begin{pmatrix} 0/\Delta & \times & \times \\ \times & \times & \times \\ \times & \times & 0/\Delta \end{pmatrix} \quad \begin{pmatrix} \times & \times & \times \\ \times & 0/\Delta & \times \\ \times & \times & 0/\Delta \end{pmatrix} \quad (5)$$

Class IV:

$$\begin{pmatrix} \times & 0/\Delta & 0/\Delta \\ 0/\Delta & \times & \times \\ 0/\Delta & \times & \times \end{pmatrix} \quad \begin{pmatrix} \times & 0/\Delta & \times \\ 0/\Delta & \times & 0/\Delta \\ \times & 0/\Delta & \times \end{pmatrix} \quad \begin{pmatrix} \times & \times & 0/\Delta \\ \times & \times & 0/\Delta \\ 0/\Delta & 0/\Delta & \times \end{pmatrix} \quad (6)$$

where "0/ Δ " at (i, j) position represents the texture zero condition $M_{ij} = 0$ and the cofactor zero condition $C_{ij} = 0$; The " \times " denotes arbitrary element. One can check that the matrices with cofactor zeros in class I are equivalent to the texture zero ones.

Choosing the first matrix of class I as an example,, we have

$$M_\nu = \begin{pmatrix} \Delta & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \times \end{pmatrix} \Rightarrow M_\nu^{-1} = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \Rightarrow M_\nu = \begin{pmatrix} \times & \times & \times \\ \times & 0 & 0 \\ \times & 0 & \times \end{pmatrix} \quad (7)$$

Thus the parallel texture structures with texture/cofactor zeros are equivalent to the no-parallel structures with two texture zeros. Although the parallel texture zero structures has been explored extensively[15, 18, 19], the analysis of the no-parallel two texture zero structure has never been reported. On the other hand, as having been pointed out in Ref.[15, 17], the class IV leads to the decoupling of a generation of lepton from mixing and thus not experimentally viable.

B. Useful notations

As we have mentioned, among the 4 classes only class I, II and III are nontrivial physical implications. In this paper we choose

$$M_{l/\nu}^I = \begin{pmatrix} 0/\Delta & \times & 0\Delta \\ \times & \times & \times \\ 0\Delta & \times & \times \end{pmatrix} \quad M_{l/\nu}^{II} = \begin{pmatrix} 0/\Delta & \times & \times \\ \times & \times & 0/\Delta \\ \times & 0/\Delta & \times \end{pmatrix} \quad M_{l/\nu}^{III} = \begin{pmatrix} 0/\Delta & \times & \times \\ \times & 0/\Delta & \times \\ \times & \times & \times \end{pmatrix} \quad (8)$$

In our analysis, we consider the charged lepton mass matrix M_l is to be Hermitian and the Majorana neutrino mass texture M_ν is complex and symmetric. Therefore M_l and M_ν are diagonalized by unitary matrix V_l and V_ν

$$M_l = V_l M_l^D V_l^\dagger \quad M_\nu = V_\nu M_\nu^D V_\nu^T \quad (9)$$

where $M_l^D = \text{Diag}(m_e, m_\mu, m_\tau)$, $M_\nu^D = \text{Diag}(m_1, m_2, m_3)$. The Pontecorvo-Maki-Nakagawa-Sakata matrix[21] U_{PMNS} is given by

$$U_{PMNS} = V_l^\dagger V_\nu \quad (10)$$

and is parameterized as

$$U_{PMNS} = U P_\nu = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix} \quad (11)$$

where we use the abbreviation $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The (α, β) in P_ν represent the two Majorana CP-violating phases and δ denotes the Dirac CP-violating phase. In order to facilitate our calculation, we treat the Hermitian matrix M_l^{-1} factorisable. i.e

$$M_l = K_l M_l^r K_l^\dagger \quad (12)$$

where K_l is the unitary phase matrix parameterized as $K_l = \text{diag}(1, e^{i\phi_1}, e^{i\phi_2})$. The M_l^r becomes a real symmetric matrix which can be diagonalized by real orthogonal matrix O_l . Then we have

$$V_l = K_l O_l \quad (13)$$

and

$$U_{PMNS} = O_l^T K_l^\dagger V_\nu \quad (14)$$

From (9), (10) and (14), the neutrino mass matrix M_ν is given by

$$M_\nu = K_l V P_\nu M_\nu^D P_\nu V^T K_l^\dagger \quad (15)$$

where $V \equiv O_l U$. From (15) and solving the cofactor zero conditions of M_ν

$$M_{\nu(pq)} M_{\nu(rs)} - M_{\nu(tu)} M_{\nu(vw)} = 0 \quad M_{\nu(p'q')} M_{\nu(r's')} - M_{\nu(t'u')} M_{\nu(v'w')} = 0 \quad (16)$$

we get

$$\frac{m_1}{m_2} e^{-2i\alpha} = \frac{K_3 L_1 - K_1 L_3}{K_2 L_3 - K_3 L_2} \quad (17)$$

$$\frac{m_1}{m_3} e^{-2i\beta} = \frac{K_2 L_1 - K_1 L_2}{K_3 L_2 - K_2 L_3} e^{2i\delta} \quad (18)$$

where

$$K_i = (V_{pj} V_{qj} V_{rk} V_{sk} - V_{tj} V_{uj} V_{vk} V_{wk}) + (j \leftrightarrow k) \quad (19)$$

$$L_i = (V_{p'j} V_{q'j} V_{r'k} V_{s'k} - V_{t'j} V_{u'j} V_{v'k} V_{w'k}) + (j \leftrightarrow k) \quad (20)$$

with (i, j, k) a cyclic permutation of $(1, 2, 3)$. With the help of Eq.(17) and (18), the magnitudes of neutrino mass ratios are given by

$$\rho = \left| \frac{m_1}{m_3} e^{-2i\beta} \right| \quad (21)$$

$$\sigma = \left| \frac{m_1}{m_2} e^{-2i\alpha} \right| \quad (22)$$

with the two Majorana CP-violating phases

$$\alpha = -\frac{1}{2} \arg \left(\frac{K_3 L_1 - K_1 L_3}{K_2 L_3 - K_3 L_2} \right) \quad (23)$$

$$\beta = -\frac{1}{2} \arg \left(\frac{K_2 L_1 - K_1 L_2}{K_3 L_3 - K_2 L_3} e^{2i\delta} \right) \quad (24)$$

The results of Eq. (21),(22), (23) and (24) imply that the two mass ratio (ρ and σ) and two Majorana CP-violating phases (α and β) are fully determined in terms of the real orthogonal matrix O_l , $U(\theta_{12}, \theta_{23}, \theta_{13}$ and δ). The neutrino mass ratios ρ and σ are related to the ratios of two neutrino mass-squared ratios obtained from the solar and atmosphere oscillation experiments as

$$R_\nu \equiv \frac{\delta m^2}{\Delta m^2} = \frac{2\rho^2(1 - \sigma^2)}{|2\sigma^2 - \rho^2 - \rho^2\sigma^2|} \quad (25)$$

and to the three neutrino mass as

$$m_2 = \sqrt{\frac{\delta m^2}{1 - \sigma^2}} \quad m_1 = \sigma m_2 \quad m_3 = \frac{m_1}{\rho} \quad (26)$$

where $\delta m^2 \equiv m_2^2 - m_1^2$ and $\Delta m^2 \equiv |m_3^2 - \frac{1}{2}(m_1^2 + m_2^2)|$. In the following numerical analysis, we utilize the recent 3σ confidential level global-fit data from the neutrino oscillation experiments[22].i.e

$$\begin{aligned} \sin^2 \theta_{12}/10^{-1} &= 3.08_{-0.49}^{+0.51} & \sin^2 \theta_{23}/10^{-1} &= 4.25_{-0.68}^{+2.16} & \sin^2 \theta_{13}/10^{-2} &= 2.34_{-0.57}^{+0.63} \\ \delta m^2/10^{-5} &= 7.54_{-0.55}^{+0.64} eV^2 & \Delta m^2/10^{-3} &= 2.44_{-0.22}^{+0.22} eV^2 \end{aligned} \quad (27)$$

for normal hierarchy (NH) and

$$\begin{aligned} \sin^2 \theta_{12}/10^{-1} &= 3.08_{-0.49}^{+0.51} & \sin^2 \theta_{23}/10^{-1} &= 4.25_{-0.74}^{+2.22} & \sin^2 \theta_{13}/10^{-2} &= 2.34_{-0.61}^{+0.61} \\ \delta m^2/10^{-5} &= 7.54_{-0.55}^{+0.64} eV^2 & \Delta m^2/10^{-3} &= 2.40_{-0.23}^{+0.21} eV^2 \end{aligned} \quad (28)$$

for inverted hierarchy(IH). By this time, no constraint is added on the Dirac CP-violating phase δ at 3σ level, however the recent numerical analysis[22] tends to give the best-fit value $\delta \approx 1.40\pi$. In neutrino oscillation experiments, CP violation effect is usually reflected by the Jarlskog rephasing invariant quantity[23] defined as

$$J_{CP} = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \delta \quad (29)$$

The Majorana nature of neutrino can be determined if any signal of neutrinoless double decay is observed, implying the violation of leptonic number violation. The decay ratio is related to the effective of neutrino m_{ee} , which is written as

$$m_{ee} = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}| \quad (30)$$

Although a 3σ result of $m_{ee} = (0.11 - 0.56)$ eV is reported by the Heidelberg-Moscow Collaboration[24], this result is criticized in Ref [25] and shall be checked by the forthcoming experiment. It is believed that that the next generation $0\nu\beta\beta$ experiments, with the sensitivity of m_{ee} being up to 0.01 eV[26], will open the window to not only the absolute neutrino mass scale but also the Majorana-type CP violation. Besides the $0\nu\beta\beta$ experiments, a more severe constraint was set from the recent cosmology observation. Recently, an upper bound on the sum of neutrino mass $\sum m_i < 0.23$ eV is reported by Plank Collaboration[27] combined with the WMAP, high-resolution CMB and BAO experiments.

III. NUMERICAL ANALYSIS

We have proposed a detailed numerical analysis for both class I, II and III. It turns out that the class II are phenomenologically acceptable only for normal hierarchy and the class I and III are ruled out for both normal hierarchy and inverted hierarchy. In this section we presented the main prediction of class II.

We start from the factorisable formation of charged lepton matrix M_l^r parameterized as

$$(M_l^r)^{II} = \begin{pmatrix} 0 & a & c \\ a & b & 0 \\ c & 0 & d \end{pmatrix} \quad (31)$$

and diagonalized by an orthogonal matrix O_l

$$O_l^T (M_l^r)^{II} O_l = \text{diag}(m_e, -m_\mu, m_\tau) \quad (32)$$

. Without losing generality, the coefficients a, c, d are set to be real and positive. One should bear in mind that the minus sign in (32) is introduced to facilitate the

analytical calculation and has no physical meaning since it originates from the phase transformation of the Dirac fermions in charged lepton sector. Following the same strategy of Ref.[15] and using the invariant $\text{Tr}(M_l)^r$, $\text{Det}(M_l)^r$ and $\text{Tr}(M_l)^{r^2}$, the nonzero elements of $(M_l)^r$ can be expressed in terms of three mass eigenvalues m_e, m_μ, m_τ and d

$$a = \sqrt{-\frac{(m_e - m_\mu - d)(m_e + m_\tau - d)(-m_\mu + m_\tau - d)}{m_e - m_\tau + m_\tau - 2d}} \quad (33)$$

$$b = m_e - m_\mu + m_\tau - d \quad (34)$$

$$c = \sqrt{\frac{(d - m_e)(d + m_\mu)(d - m_\tau)}{m_e - m_\tau + m_\tau - 2d}} \quad (35)$$

where the parameter d is allowed in the range of $0 < d < m_e$ and $m_\tau - m_\mu < d < m_\tau$. Then the O_l can be easily constructed as

$$O_l = \begin{pmatrix} \frac{(b-m_e)(d-m_e)}{N_1} & \frac{(b+m_\mu)(d+m_\mu)}{N_2} & \frac{(b-m_\tau)(d-m_\tau)}{N_3} \\ -\frac{a(d-m_e)}{N_1} & -\frac{a(d+m_\mu)}{N_2} & -\frac{a(d-m_\tau)}{N_3} \\ -\frac{c(b-m_e)}{N_3} & -\frac{c(b+m_\mu)}{N_3} & -\frac{c(b-m_\tau)}{N_3} \end{pmatrix} \quad (36)$$

where N_1, N_2 and N_3 are the normalized coefficients given by

$$N_1^2 = (b - m_e)^2(d - m_e)^2 + a^2(d - m_e)^2 + c^2(b - m_\tau)^2 \quad (37)$$

$$N_2^2 = (b + m_\mu)^2(d + m_\mu)^2 + a^2(d + m_\mu)^2 + c^2(b + m_\mu)^2 \quad (38)$$

$$N_3^2 = (b - m_\tau)^2(d - m_\tau)^2 + a^2(d - m_\tau)^2 + c^2(b - m_\tau)^2 \quad (39)$$

Replacing the (21), (22), (23), (24) and (25) with the O_l obtained in (36), we can see that the ratios of mass (ρ, σ) , two Majorana CP-violating phases (α, β) and the ratio of mass squared difference R_ν can be expressed via eight parameters: three mixing angle $\theta_{12}, \theta_{23}, \theta_{13}$, one Dirac CP violating phase δ , three charged lepton mass (m_e, m_μ, m_τ) and the parameter d . Here we choose the three charged lepton mass at the electroweak scale ($\mu \simeq M_Z$) i.e[28]

$$m_e = 0.486570154 \text{ MeV} \quad m_\mu = 102.7181377 \text{ MeV} \quad m_\tau = 1746.17 \text{ MeV} \quad (40)$$

In the numerical analysis, a set of random numbers are generated for the three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and mass square differences $(\delta m^2, \Delta m^2)$ in their 3σ range.

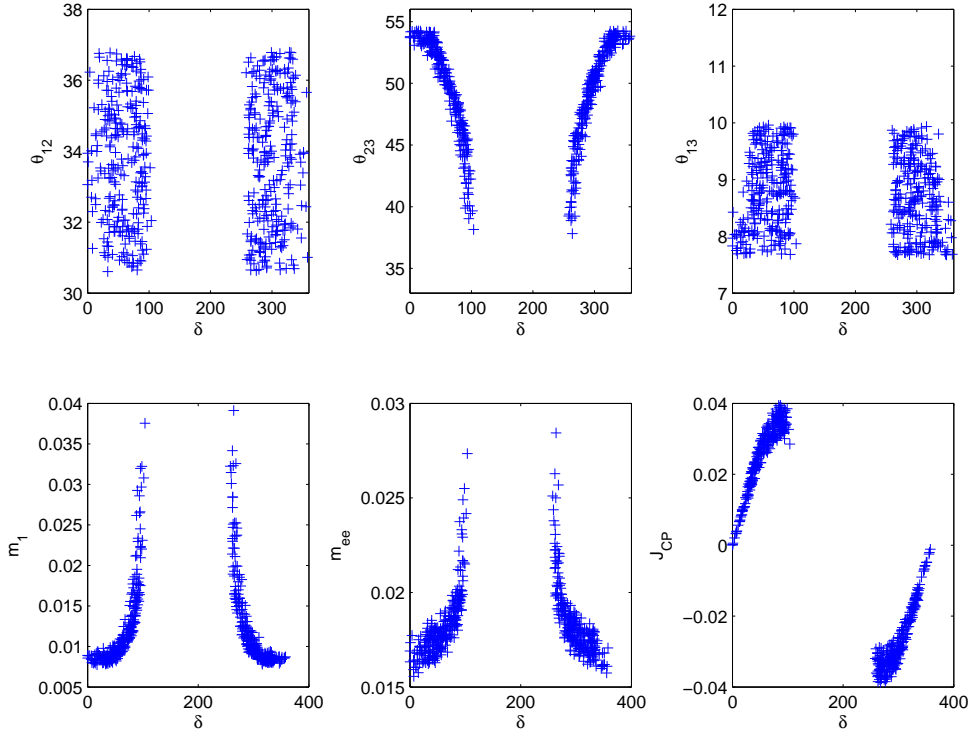


Figure 1: The correlation plots for class II(NH).

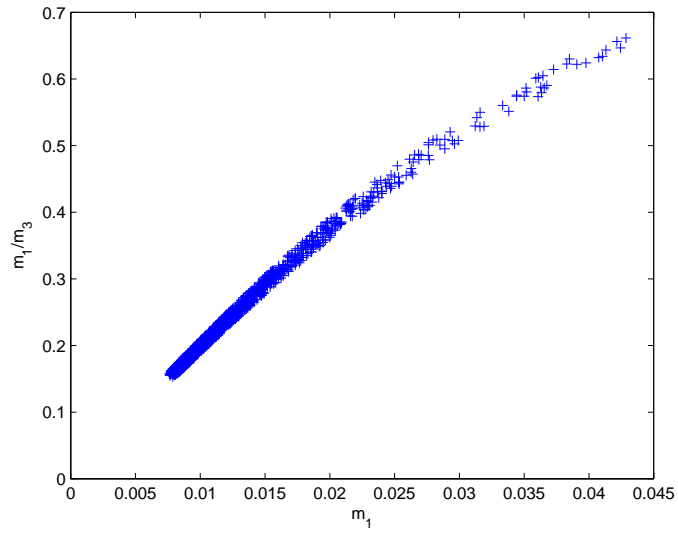


Figure 2: The ratio of $\frac{m_1}{m_3}$ as a function of m_1 (eV)

We also randomly vary the parameter d in its appropriate range. Since at 3σ level the Dirac CP-violating phase δ is unconstrained in neutrino oscillation experiments, we vary it randomly in the range of $[0, 2\pi)$. With the random number and using Eq. (21), (22) and (25), neutrino mass ratios (ρ, σ) and the mass-squared difference ratio R_ν are determined. Then the input parameters is empirically acceptable when the R_ν falls inside the the 3σ range of experimental data, otherwise they are ruled out. Finally, we get the value of neutrino mass and Majorana CP-violating α and β though Eq.(23), (24) and (26). Once the the absolute neutrino mass $m_{1,2,3}$ are obtained , the further constraint from cosmology should be considered. In this paper, the upper bound on the sum of neutrino mass Σm_i is set to be less than 0.23 eV.

The numerical results of class II for normal hierarchy are presented in Fig.1. We can see from the figures that the three neutrino mixing angle θ_{12} , θ_{23} , and θ_{13} vary arbitrarily in its 3σ range while the Dirac CP-violated phase δ are restricted to range of $\delta < 100^\circ$ and $\delta > 260^\circ$. Moreover, there exhibits a strong correlation between δ and θ_{23} . For instance, if $\delta \approx 100^\circ$ or 260° holds, the θ_{23} appears to be less then 45° . This is particularly interesting since the recent global fit trends to give the $\theta_{23} < 45^\circ$ at 2σ level. It is believed that the strong $\delta - \theta_{23}$ correlation predicted here is essential for the model selection and will be confirmed or ruled out by future long-baseline neutrino oscillation experiments. The similar correlations also holds between δ , m_{ee} and the lightest neutrino mass m_1 . Especially, there exists a highly constrained range of $0.080\text{eV} < m_1 < 0.045\text{eV}$. This leads to a mild neutrino mass hierarchy which can be seen from the Figure. 2. The effective Majorana neutrino mass m_{ee} are highly constrained in the range of $0.015\text{eV} < m_{ee} < 0.095\text{eV}$, which reaches the accuracy of the future neutrinoless double beta decay ($0\nu\beta\beta$) experiments. We also observed that the allowed range of Jarlskog rephasing invariant $|J_{CP}|$ is $0 \sim 0.04$, however if $\delta \approx 100^\circ$ or 260° holds, one obtain the $|J_{CP}| > 0.03$ which is potentially detected by future long-baseline neutrino oscillation experiments.

IV. THE $Z_4 \times Z_2$ FLAVOR SYMMETRY REALIZATION

In this section, we show that the lepton mass matrix of class II can be realized based on the type-I seesaw models with the $Z_4 \times Z_2$ Abelian flavor symmetry. It has been proved that the texture/cofactor zeros in mass matrices can be obtained from Z_n symmetry[7–9]. In this paper, we take the same strategy. It is noted that in the flavor basis, the neutrino mass matrix belonging to class II is realized under Z_8 symmetry[7]. Different from Ref.[7], we build the model under the basis where M_l is nodiagonal. Under the $Z_4 \times Z_2$ symmetry, the three charged lepton doublets $D_{iL} = (\nu_{iL}, l_{iL})$, three right-handed charged lepton singlets l_{iR} and three right-handed neutrinos ν_{iR} (where $i = e, \mu, \tau$) transform as

$$\begin{aligned} \nu_{eR} &\sim (\omega, 1), & \nu_{\mu R} &\sim (1, 1), & \nu_{\tau R} &\sim (\omega^2, 1) \\ D_{eL} &\sim (\omega, -1), & D_{\mu L} &\sim (1, -1), & D_{\tau L} &\sim (\omega^2, -1) \\ l_{eR} &\sim (\omega^3, -1), & l_{\mu R} &\sim (1, -1), & l_{\tau R} &\sim (\omega^2, -1) \end{aligned} \quad (41)$$

where $\omega = e^{i\pi/2}$. Then the bilinears of $\overline{D}_{iL}l_{jR}$, $\overline{D}_{iL}\nu_{jR}$, and $\nu_{iR}^T\nu_{jR}$, transform respectively under Z_4 as

$$\begin{pmatrix} -1 & -i & i \\ -i & 1 & -1 \\ i & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -i & i \\ -i & 1 & -1 \\ i & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & i & -i \\ i & 1 & -1 \\ -i & -1 & 1 \end{pmatrix} \quad (42)$$

To generate the fermion mass, we need introduce the three Higgs doublets $\Phi_{12}, \Phi_{23}, \Phi$ for charged lepton matrix M_l , one the Higgs doublet Φ' for Dirac neutrino mass matrix M_D and a scalar singlet χ for the Majorana neutrino mass matrix M_R , which transform under $Z_4 \times Z_2$ symmetry as

$$\begin{aligned} \Phi_{12} &\sim (\omega, 1), & \Phi_{13} &\sim (\omega^3, 1), & \Phi &\sim (1, 1) \\ \Phi' &\sim (1, -1), & \chi &\sim (\omega, 1) \end{aligned} \quad (43)$$

To maintain the the Yukawa Lagrange is invariant under the flavor symmetry transformation, one can see from (42) that the Φ_{12} and Φ_{13} couple to the bilinears $\overline{D}_{eL}l_{\mu R}$ and $\overline{D}_{eL}l_{\tau R}$ to produce the (1,2) and (1,3) nonzero matrix elements in M_l ; Φ couples

to $\overline{D}_{\mu L} l_{\mu R} \overline{D}_{\tau L} l_{\tau R}$ to produce the (2,2) and (3,3) no zero matrix elements. The zero matrix elements in M_l is obtained because there are no appropriate scalars to generate them. For the Dirac neutrino mass sector, since there exists only one scalar doublet Φ' which is invariant under Z_4 , the Φ' will contribute to the (1,1), (2,2), (3,3) elements and make the M_D to be diagonal. Here the Z_2 symmetry is used to distinguish the set of scalar doublets $(\phi_{12}, \phi_{13}, \phi)$ and ϕ' so that they are respectively in charge of the mass generation of M_l and M_D without any crossing. In order to produce the Majorana neutrino mass term, we introduce a complex scalar singlet χ . The χ couples to $\nu_{eR}^T \nu_{\tau R}$ while χ^* couples to $\nu_{eR}^T \nu_{\mu R}$, leading to the (1,2) and (1,3) no zero elements in M_R . From (42), the $\nu_{\mu R}^T \nu_{\mu R}$ and $\nu_{\tau R}^T \nu_{\tau R}$ is invariant under Z_4 , thus we can directly write them in the Lagrange without needing the singlet χ_{22} and χ_{33} . As the same of M_l and M_D , we obtain the (1,1),(2,3) zero matrix elements in M_R by not introducing other scalar singlets. Therefore, the mass matrices M_l , M_D and M_R is given by

$$M_l \sim \begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix} \quad M_D \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix} \quad M_R \sim \begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix} \quad (44)$$

Using the neutrino mass formula of type-I seesaw mechanism $M_\nu = -M_D^T M_R M_D$, we obtain

$$M_\nu \sim \begin{pmatrix} \Delta & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \times \end{pmatrix} \quad (45)$$

Together with the M_l in (44), we have realized the parallel texture/cofactor zeros leptonic mass matrix of class II under the $Z_4 \times Z_2$ flavor symmetry.

V. CONCLUSION AND DISCUSSION

We have investigated the parallel texture structures with texture zeros in lepton mass matrix M_l and neutrino mass matrix M_ν and studied their phenomenological implications. These matrices can not obtained from arbitrary Hermitian texture by

making WB transformations. The 15 possible textures are grouped into class I, II, III, and IV, where the matrices in each class are related by means of permutation transformation and share the same physical implications. We found only class I, II, III are not trivial by the fact that the matrices of class IV is obviously not phenomenologically viable. A detailed numerical analysis for class I, II and III is proposed. It is shown that only normal hierarchy pattern of class II are allowed by the recent global fit experimental data. We demonstrate the correlation plots between Dirac CP-violating phase δ , three mixing angles θ_{12} , θ_{23} and θ_{13} , the effective neutrino mass m_{ee} , lightest neutrino mass m_1 and the ratio of m_1/m_3 , leading to the predictions to be confirmed by future experiments. A realization of the model base on $Z_4 \times Z_2$ flavor symmetry is illustrated.

Finally we mentioned that in the spirit of Ref. [15, 16], the parallel texture structures are treated as a natural precursor of more general cases. A systematic analysis of all possible combinations with texture/cofactor zeros in M_l/M_ν and the flavor symmetry realization will be published in [29].

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